



# Electrostatic field theory and circuit analysis in the design of coalescers with pulsed dc voltage

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## ABSTRACT

This paper presents a mathematical analysis of the electric field and interfacial free charge experienced by the emulsion in an electrostatic coalescer that is subjected to a pulsed dc voltage. Previous work in this area is re-assessed, but the analysis is expanded. A novel static analysis of the system is presented, giving valuable physical insight. It becomes obvious that there must be a limited range of frequencies that are effective in a pulsed dc scheme. Thereafter, analytical expressions for the steady-state solution, where both the switching frequency and duty ratio (or mark space ratio) can be varied, are presented for the first time. These expressions are used to derive voltage and charge profiles for a specific example under various circumstances. In addition, a previously found criterion for optimum switching frequency is analyzed, and it is found that there are actually three physical parameters that all would show the same correlation with coalescer performance, not only charge as previously found, but also voltage and current. An analytical expression for the correlation quantity is derived, and the numerical example indicates that there is a narrow range of frequencies and duty ratios that are effective.

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## 1. Introduction

The oil extracted from offshore fields normally has high water content. Much is free water, which can be separated quite easily by cyclones or gravitational settlers [1]. However, after this initial step, a water-in-oil emulsion is formed. As a bulk substance the emulsion has high resistivity, and therefore a high intensity electric field can be applied to further enhance separation through coalescence of smaller water droplets.

The two most important mechanisms that take place are probably dipole- and migratory coalescence. The latter is a consequence of movement of charged water droplets under the influence of the electric field; the droplets may have been charged by ions or free electrons present in the emulsion. Dipole coalescence happens because of polarization of the water droplets.

A lot of work has been performed on electrostatic coalescence over the years. The present status has been extensively described elsewhere [2,3]. However, the design of coalescers is notoriously difficult and requires a multi-disciplinary approach. This is becoming increasingly true since the present trend of thought in the oil-industry is to make coalescers more compact, and place them upstream in the process chain [4]. There seems to be a lack of clear design criteria for such components, which hampers their

evolution. This raises a requirement for a better understanding of both the fundamental physics of the micro-scale particle attractions and the nature of the electric field that is actually experienced by the emulsion, and thereby the water droplets, under various circumstances. Lately, work has been performed to address the first of these issues both through experiments and numerical means [5,6].

This paper addresses the other of these issues, that of the overall electric field. Here, the electric field is analyzed under a scheme where the coalescer is subjected to a pulsed dc voltage. In such a situation, experiments show that there is a frequency dependence of coalescer behavior [7,8]. This has previously been explained by modeling the coalescer as a two-layer lossy dielectric, where it was found that the optimum frequency correlated with the maximum mean square of the variable part of the interfacial free charge in the off-period [9,10].

In this work the two-layer capacitor model is retained, but the analysis is expanded. Electrostatic theory is first used in order to gain physical insight into the build-up of charge and voltage in the system. Furthermore, circuit analysis is used for a dynamic view of the system, and this leads to the insight that there are actually three physical parameters that correlate with the optimum frequency (assuming previous work to be correct)—not only charge, but also the mean square of (the variable parts of) voltage and conductive current in the off-period. This seems not to have been mentioned in previous work, but could have some significance.

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## 2. Short review of the two-layer capacitor model

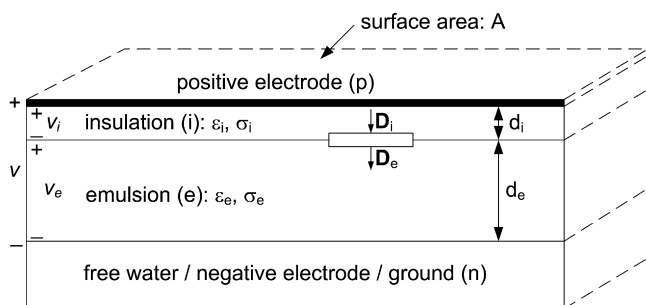
Bailes and Larkai carried out laboratory experiments on the optimization of coalescence with a device that was rectangular in cross-section. Since its length and width was significantly larger than its thickness, the geometry was essentially two-dimensional. The results from the experiments were published in papers dating back to 1981 [7,8]. It was found that the success of coalescence depended on the frequency of the applied pulsed dc voltage. The optimum frequency depended on the conductivity and permittivity and the dimensions of the insulation and emulsion.

In 1985 an electrical model of the system used by Bailes and Larkai was proposed by Joos and Snaddon [9]. They modeled the coalescer as a two-layer capacitor, and used an appropriate model for the power supply and switching circuitry. They solved the circuit, and argued that the mean square of the voltage experienced by the emulsion should be maximized for best coalescer performance. However, their optimum frequency turned out to be 22 Hz for the particular system that had been used, whereas the experimentally found optimum was 8 Hz. The discrepancy was explained by uncertainties in the modeling of the emulsion.

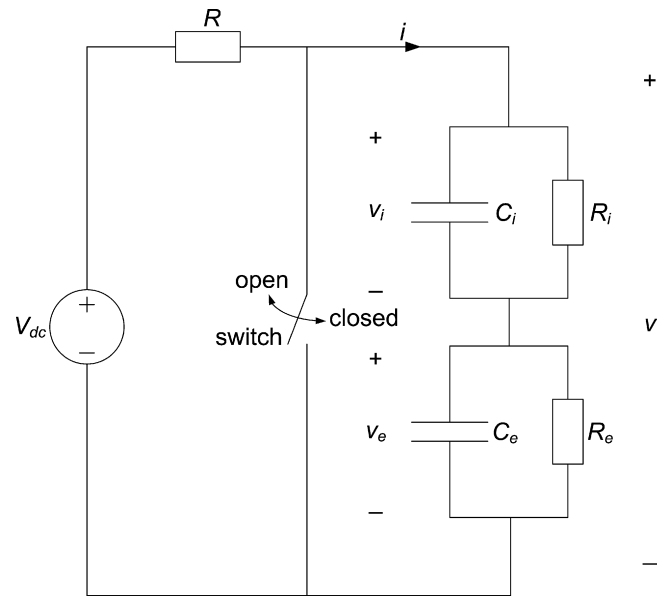
Later, in 1995, Bailes used the model of Joos and Snaddon to come up with a new explanation of the experimental findings [10]. He proposed that it could be the mean square of the variable part of the surface charge density at the interface between the emulsion and the insulation in the off-period that was the decisive parameter. This quantity had a distinct maximum at precisely the experimentally found optimum.

The experimental system used had a positive electrode that consisted of a flat metal plate covering the whole upper surface of the coalescer. Then there was an insulating layer (dielectric material) between the electrode and the emulsion. The free water in the oil separated easily and formed a third layer at the bottom. The free water was in direct contact with the ground electrode, and was therefore seen as part of ground. Fig. 1 schematically depicts the cross-section of the system. The global material properties (permittivity  $\epsilon$  and conductivity  $\sigma$ ) that are indicated in the figure are well defined for the insulation material. For the emulsion, these parameters are not given once and for all; instead, they may change both in the short- and long term since they will depend micro-scale properties of the emulsion (such as droplet diameters, droplet velocity, droplet distribution, zeta potential and so on). However, it is reasonable to assume that the emulsion can be considered homogenous enough for the global material properties to remain constant within the time-frame for the analyses that are presented below.

Fig. 2 depicts the resulting two-layer capacitor model. The parameters can, due to the geometry of the particular coalescer



**Fig. 1.** Schematic representation of the system cross-section. Its three-dimensional nature is indicated with dashed lines. The global material properties and the geometrical parameters that are used in the mathematical derivations are indicated. From these parameters, the resistance and the capacitance of each layer can be calculated, with the circuit model of Fig. 2 as a result. A Gaussian surface at the interface between insulation and emulsion is also shown, indicating the manner in which Gauss' law is applied.



**Fig. 2.** The two-layer capacitor model of the electrostatic coalescer, and its power supply. Subscript *i* denotes insulation, whereas *e* is emulsion. The switch in the model of the power supply changes state periodically from open, which corresponds to the on-period, to closed, which corresponds to the off-period.

discussed, be found from the standard formulas for a parallel plate capacitor. It should be noted that the modeling of the emulsion is not trivial since it is actually a flowing liquid. However, for the electric field to have an effect at all, it must be assumed that the substance stays inside the coalescer for a sufficiently long period of time. It is referred to the original papers for further discussions on the validity of the model itself; here we shall take it for granted.

The power supply, also depicted in Fig. 2, is connected to the terminals of the coalescer, and it is modeled as a voltage source in series with a resistance, and with a switch in parallel which periodically changes state from open to closed. The latter situation corresponds to the off-period, and the open state to the on-period, indicating that the source voltage is applied on the coalescer terminals.

## 3. Analysis using electrostatic theory

### 3.1. Application of step voltage

In order to gain physical insight, the system is analyzed from a static perspective in this section. We assume here that the resistance  $R$  is zero, such that when the switch is open, the voltage  $V_{dc}$  is directly applied on the terminals of the coalescer. Similarly, when the switch closes the short circuit is also applied without any time-delay. Between the state changes, we assume a sufficiently long period of time such that a static situation is arrived at both in the on- and off-periods.

When a step voltage is applied in this manner across a capacitor, the electrodes are charged instantaneously. This implies that the current in the external circuit is a dirac-pulse, infinite in value, but infinitesimally short in duration. Similarly, when a capacitor is short circuited in this manner, a dirac-pulse of current redistributes the charge on the electrodes instantaneously. This is a situation one would try to avoid in a physical system as the stress on the components in the circuit could be destructive. However, this kind of analysis gives important physical insight, and we will therefore assume that the voltage profile of  $v$  (the voltage applied on the coalescer terminals) is as depicted in Fig. 3.

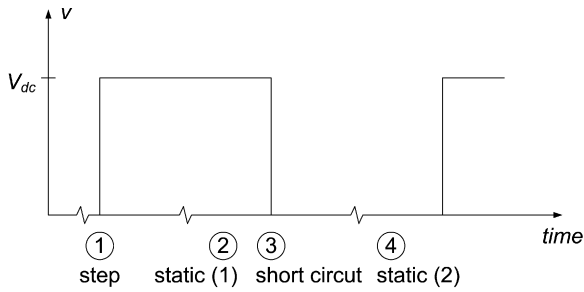


Fig. 3. Idealized voltage profile applied on the terminals (electrodes) of the coalescer.

In the dynamic analysis presented later, the value of  $R$  will be assumed non-zero, and a time-delay in the charging of the electrodes will therefore result. However, the short circuit will be applied in an instantaneous manner also in the dynamic analysis. It is possible, and probably more physically correct, to include a resistance also in the short-circuit branch, but since the paper includes a discussion of previously obtained results, the original model is retained.

### 3.2. Basic equations

Gauss' law, one of Maxwell's four equations, states that

$$\oint_S \mathbf{D} \cdot d\mathbf{S} = Q \quad (1)$$

$Q$  is free charge enclosed by the surface  $S$ , whereas  $\mathbf{D}$  is the electric displacement, which is related to the electric field  $\mathbf{E}$  via the constitutive equation:

$$\mathbf{D} = \varepsilon \mathbf{E} \quad (2)$$

$\varepsilon = \varepsilon_0 \varepsilon_r$  is the permittivity of the material, given as the product of the permittivity of free space ( $\varepsilon_0 \approx 8.854 \times 10^{-12} \text{ F m}^{-1}$ ) and the relative permittivity of the material in question ( $\varepsilon_r$ ).

In a static situation, the electric field is further related to the electrostatic potential  $V$  through:

$$\mathbf{E} = -\nabla V \quad (3)$$

In a situation such as the one modeled here, the electric field can be considered homogenous in both the two substances. This implies that the electric field is simply the voltage across the substance divided by its width. Using the expression for the capacitance of a parallel plate capacitor, the electric displacement in the two substances can now be written:

$$D_i = \varepsilon_i E_i = \varepsilon_i \frac{v_i}{d_i} = \frac{C_i}{A} v_i \quad (4)$$

$$D_e = \varepsilon_e E_e = \varepsilon_e \frac{v_e}{d_e} = \frac{C_e}{A} v_e \quad (5)$$

Using Gauss' law, Eq. (1), we can now find an expression for the free charge,  $Q_{e-i}$ , at the emulsion–insulation interface:

$$Q_{e-i} = (D_e - D_i)A = C_e v_e - C_i v_i \quad (6)$$

The law of conservation of charge can be expressed as

$$\nabla \cdot \mathbf{J} = -\frac{\partial \rho}{\partial t} \quad (7)$$

where  $\mathbf{J}$  is conduction current density and  $\rho$  is free charge density. Consequently, when a static situation exists:

$$\nabla \cdot \mathbf{J} = 0 \quad (8)$$

This can equivalently be expressed as

$$\oint_S \mathbf{J} \cdot d\mathbf{S} = 0 \quad (9)$$

Furthermore, the conduction current density is related to the electric field via:

$$\mathbf{J} = \sigma \mathbf{E} \quad (10)$$

$\sigma$  is conductivity. Following a similar line of reasoning as above, we find that

$$J_i = \sigma_i E_i = \sigma_i \frac{v_i}{d_i} = \frac{v_i}{R_i A} \quad (11)$$

$$J_e = \sigma_e E_e = \sigma_e \frac{v_e}{d_e} = \frac{v_e}{R_e A} \quad (12)$$

Then, using Eq. (9), we find that:

$$\frac{v_e}{R_e} - \frac{v_i}{R_i} = 0 \quad (13)$$

### 3.3. Application of step voltage—initial situation

If the coalescer is initially uncharged, and a step voltage  $v = V_{dc}$  is then applied to its terminals (the instant denoted “step” in Fig. 3), the voltage distribution over the substances (insulation and emulsion) will initially be given by their capacitances. This is because it takes time for free charge to migrate through the physical substances to accumulate at the interface. Consequently,  $Q_{e-i}$  is initially zero, such that Eq. (6) results in

$$C_e v_e = C_i v_i \quad (14)$$

Since, further,  $v = v_i + v_e$ , it follows that

$$v_i = \frac{C_e}{C_i + C_e} V_{dc} \quad \text{and} \quad v_e = \frac{C_i}{C_i + C_e} V_{dc} \quad (15)$$

### 3.4. Application of constant voltage—final situation

If the two substances were ideal dielectrics (zero conductivity), Eq. (15) would continue to describe the voltage distribution. Here, however, the non-zero conductivities strongly influence the situation since free charge starts to flow due to the applied field. If the applied voltage is on (and constant) for a sufficiently long time, a static situation is eventually achieved such that Eq. (13) is valid. This corresponds to the instant denoted “static (1)” in Fig. 3. Since it is still true that  $v = v_i + v_e$ , it follows that

$$v_i = \frac{R_i}{R_i + R_e} V_{dc} \quad \text{and} \quad v_e = \frac{R_e}{R_i + R_e} V_{dc} \quad (16)$$

The final interfacial free charge can then be calculated with Eq. (6) to be

$$Q_{e-i} = \frac{C_e R_e - C_i R_i}{R_i + R_e} V_{dc} \quad (17)$$

Thus, depending on the material properties and the geometry (and thereby the values of resistance and capacitance), a significant interfacial charge has been built up from initially zero.

### 3.5. Application of short circuit

If the electrostatic coalescer is now short circuited ( $v = 0$ ) from the static situation above (the instant denoted “short circuit” in Fig. 3), the charge  $Q_{e-i}$  will – again – initially be unchanged since to change it, free charges have to move through the substances, which takes time. It is still true that  $v = v_i + v_e$ , such that  $v_i = -v_e$ . Since the interfacial charge is known through Eq. (17), Eq. (6) can now

**Table 1**

The table shows charges and voltages when the voltage profile of Fig. 3 is applied on the terminals of the coalescer. The simplifying assumption  $R_i \rightarrow \infty$  is used in order to arrive at expressions that are easy to interpret. (The full expressions are, however, easy to derive following the methodology above.) The annotations in the first row refer to the instants in Fig. 3.

	1: step	2: static (1)	3: short circuit	4: static (2)
$Q_{i-p}$	$\frac{C_i}{C_i+C_e} C_e V_{dc}$	$C_i V_{dc}$	$\frac{C_i}{C_i+C_e} C_i V_{dc}$	0
$Q_{e-i}$	0	$-C_i V_{dc}$	$-C_i V_{dc}$	0
$Q_{n-e}$	$\frac{-C_i}{C_i+C_e} C_e V_{dc}$	0	$\frac{C_i}{C_i+C_e} C_e V_{dc}$	0
$v$	$V_{dc}$	$V_{dc}$	0	0
$v_i$	$\frac{C_e}{C_i+C_e} V_{dc}$	$V_{dc}$	$\frac{C_i}{C_i+C_e} V_{dc}$	0
$v_e$	$\frac{C_i}{C_i+C_e} V_{dc}$	0	$\frac{-C_i}{C_i+C_e} V_{dc}$	0

be used to find the initial voltages after the short circuit has been applied:

$$v_i = \frac{C_i R_i - C_e R_e}{(R_i + R_e)(C_i + C_e)} V_{dc} \quad \text{and} \quad v_e = \frac{C_e R_e - C_i R_i}{(R_i + R_e)(C_i + C_e)} V_{dc} \quad (18)$$

As the short circuit lasts, the device is gradually de-charged, and after a sufficiently long period of time both  $v_i$  and  $v_e$  as well as the interfacial charge will be zero.

### 3.6. Summary—static view

Similar analysis to that above can be performed for the other two interfaces of the coalescer: the interface between the insulation and the positive electrode, and the interface between ground and the emulsion. In an actual coalescer design, it is likely that the resistance of the insulation will be much higher than that of the emulsion. If we assume an ideal dielectric ( $R_i \rightarrow \infty$ ), the expressions for charges and voltages are easier to interpret without having to resort to a numerical example. The full results with this simplifying assumption are presented in Table 1.  $Q_{i-p}$  is insulation-positive electrode interfacial charge, whereas  $Q_{n-e}$  is for ground-emulsion.

Valuable physical insight is gained from this analysis: when the voltage pulse is first applied, the voltage distributes itself over the two layers, depending on their capacitances. It is reasonable to assume that the insulation has significantly higher capacitance than the emulsion. If so, most of the applied voltage will ini-

tially be distributed across the emulsion. This implies that the electric field will now be directed from the emulsion–insulation interface towards the negative electrode, such that negative free charge will move in direction interface, and positive in direction ground electrode. Therefore, negative free charge accumulates on the emulsion–insulation interface. As time passes, the voltage distribution changes, and eventually the voltage across the emulsion is zero. At this point, no further charge separation takes place.

Then the short circuit is applied. The emulsion is immediately subjected to a negative voltage which is almost as large as  $V_{dc}$ , assuming the capacitance of the insulation dominates. The electric field in the emulsion will now have direction from ground electrode towards emulsion–insulation interface, such that negative free charge will flow in direction ground, and positive charge in the opposite direction. Eventually, therefore, the interface is de-charged, and a stationary situation is arrived at where there is no more charge separation, and zero voltage over both layers.

### 3.7. Significance of frequency

From this analysis it becomes obvious that there is a frequency dependence of coalescer behavior. If the frequency is too low, there will be significant periods where nothing interesting happens in the emulsion since the electric field will be zero both in the end of the on- and the off-periods. Thus, state change of the switch should take place well before the electric field becomes zero. However, migratory coalescence implies movement of free charge, and therefore state change should be slow enough to allow for sufficient charge separation and movement; in other words, too high a frequency should be avoided as well.

As previously mentioned, previous work has concluded with the existence of an optimum frequency. The static analysis cannot provide an answer to what that frequency would be. Therefore, a dynamic analysis of the system is required. For this purpose, circuit analysis of the system is a powerful tool.

## 4. Circuit analysis of the system

### 4.1. On-period

Kirchhoff's current law demands that the total current  $i$  flowing in the external circuit must flow through both layers of the coalescer (see Fig. 2), such that

$$i = C_i \frac{dv_i}{dt} + \frac{v_i}{R_i} = C_e \frac{dv_e}{dt} + \frac{v_e}{R_e} \quad (19)$$

Kirchhoff's voltage law yields:

$$-V_{dc} + R_i + v_i + v_e = 0 \quad (20)$$

Combining these two equations, gives us a differential equation for the voltage across the emulsion:

$$RC_i C_e \frac{d^2 v_e}{dt^2} + \left[ \left( \frac{R}{R_e} + 1 \right) C_i + \left( \frac{R}{R_i} + 1 \right) C_e \right] \frac{dv_e}{dt} + \frac{R + R_i + R_e}{R_i R_e} v_e = \frac{V_{dc}}{R_i} \quad (21)$$

The solution is

$$v_e = \frac{R_e}{R + R_i + R_e} V_{dc} + k_1 e^{-t/\tau_1} + k_2 e^{-t/\tau_2} \quad (22)$$

The time constants  $\tau_1$  and  $\tau_2$  are given by the negative inverse of the roots of the characteristic equation resulting from Eq. (21), and they are

$$\tau_{1,2} = \frac{-2RC_i C_e}{-[(R/R_e) + 1]C_i + [(R/R_i) + 1]C_e \pm \sqrt{[(R/R_e) + 1]C_i + [(R/R_i) + 1]C_e)^2 - 4RC_i C_e((R + R_i + R_e)/(R_i R_e))}} \quad (23)$$

The combination of Eqs. (19), (20) and (22) gives us the solution also for  $v_i$ :

$$v_i = \frac{R_i}{R + R_i + R_e} V_{dc} + \left( \frac{RC_e}{\tau_1} - \frac{R + R_e}{R_e} \right) k_1 e^{-t/\tau_1} + \left( \frac{RC_e}{\tau_2} - \frac{R + R_e}{R_e} \right) k_2 e^{-t/\tau_2} \quad (24)$$

The constants  $k_1$  and  $k_2$  appear in both  $v_e$  and  $v_i$ . If the coalescer is initially uncharged (such that both voltages are zero), the constants are found to be

$$k_1 = -V_{dc} \frac{\tau_2}{\tau_2 - \tau_1} \left[ \frac{\tau_1}{RC_e} + \left( \frac{\tau_2 - \tau_1}{\tau_2} - 1 \right) \frac{R_e}{R + R_i + R_e} \right] \quad (25)$$

$$k_2 = V_{dc} \frac{\tau_2}{\tau_2 - \tau_1} \left( \frac{\tau_1}{RC_e} - \frac{R_e}{R + R_i + R_e} \right) \quad (26)$$

To use these constants in the steady-state solution would be incorrect, unless the frequency was so low that we went into the

static phase of the off-period (see Fig. 3). We aim for a more general solution, and this requires that we find the constants for a situation in which  $v_i$  and  $v_e$  are not zero when the on-period commences. In order to do this, it is necessary to solve the circuit for the off-period first.

#### 4.2. Off-period

In the off-period, the coalescer is short circuited, such that

$$v_i = -v_e \quad (27)$$

It is still true that the current that flows into one terminal of the coalescer equals the current that flows out of the other. Thus,

$$C_i \frac{dv_i}{dt} + \frac{v_i}{R_i} = C_e \frac{dv_e}{dt} + \frac{v_e}{R_e} \quad (28)$$

The differential equation for  $v_e$  then becomes:

$$\frac{dv_e}{dt} = -v_e \frac{R_e + R_i}{(C_i + C_e)R_i R_e} = -\frac{v_e}{\tau_{OFF}} \quad (29)$$

$\tau_{OFF}$  is the relaxation time constant, and can also be expressed in terms of the material properties and thicknesses of the substances (using the formulas for resistance and capacitance of a parallel plate capacitor):

$$\tau_{OFF} = \frac{(C_i + C_e)R_i R_e}{R_e + R_i} = \frac{\varepsilon_i d_i + \varepsilon_e d_e}{\sigma_i d_e + \sigma_e d_i} \quad (30)$$

The solution of Eq. (29) is

$$v_e = v_{e0} e^{-t/\tau_{OFF}} \quad (31)$$

where  $v_{e0}$  is the initial value of  $v_e$  just after the short circuit has been applied.

To find  $v_{e0}$ , we use the fact that the interfacial charge  $Q_{e-i}$  does not change instantaneously; in other words, it must be the same just before and just after the short circuit. We denote these two instants  $0^-$  and  $0^+$ , respectively, and use Eq. (6) to set up the following expression, where we have also introduced the superscripts ON and OFF on the voltages to distinguish between the solutions found for the on- and off-periods:

$$C_e v_e^{OFF}(0^+) - C_i v_i^{OFF}(0^+) = C_e v_e^{ON}(0^-) - C_i v_i^{ON}(0^-) \quad (32)$$

Eq. (27) is valid in the whole off-period, such that

$$v_i^{OFF}(0^+) = -v_e^{OFF}(0^+) \quad (33)$$

and therefore,

$$v_e^{OFF}(0^+) = v_{e0} = \frac{C_e v_e^{ON}(0^-) - C_i v_i^{ON}(0^-)}{C_e + C_i} \quad (34)$$

Now the question of finding  $v_{e0}$  is transformed to the task of finding the final values of  $v_e$  and  $v_i$  in the on-period, which brings us back to finding  $k_1$  and  $k_2$  for the solution in the on-period, and this insights helps us wrapping it all together in the steady-state solution, meaning that we have a state where the physical quantities repeat themselves periodically.

#### 4.3. Steady-state solution

At this point, we introduce the duty ratio  $D$ , which is the ratio of time for applied pulse to the period  $T_s$  Fig. 4 clarifies. With this definition, the following expressions are valid:

$$t_{on} = DT_s = \frac{D}{f_s} \quad (35)$$

$$t_{off} = T_s - t_{on} = (1 - D)T_s = \frac{1 - D}{f_s} \quad (36)$$

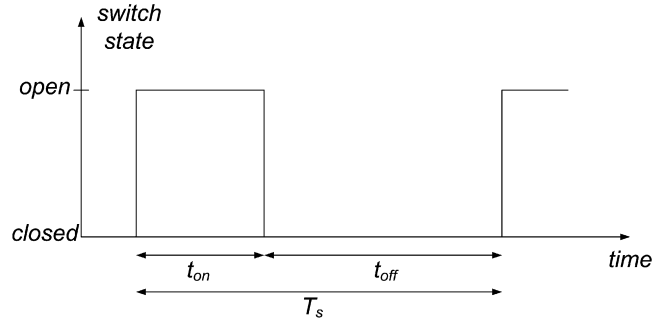


Fig. 4. In the on-period, the switch is open, whereas in the off-period, the switch is closed, short circuiting the device. The analysis is general in that the on- and off-periods must not be of the same length. For this purpose, the duty ratio  $D$  is introduced.

$f_s$  is the switching frequency ( $f_s = 1/T_s$ ). Previously, it has been reported that a  $D$  of 0.5 (mark space ratio of 1) is the optimum in a pulsed dc scheme (experimentally found) [10], but in this analysis, we aim for a more general solution of the circuit.

With these definitions, we can write the final value of the voltage in the off-period the following way:

$$v_e^{OFF}(t_{off}) = \frac{C_e v_e^{ON}(0^-) - C_i v_i^{ON}(0^-)}{C_e + C_i} e^{-(1-D)/f_s \tau_{OFF}} \quad (37)$$

Only when a short-circuit step voltage is enforced directly on the terminals of the coalescer, will the voltages across either of the layers change in a step. In the scheme that is actually used, there is a resistance between the voltage source and the coalescer, and therefore the voltages  $v_e$  and  $v_i$  will not change instantaneously when we go from the off- to the on-period. In other words, the initial values of these two voltages in the on-period will equal the final values in the off-period. If we move the instant  $t=0$  to the beginning of the on-period, we can then write:

$$\begin{aligned} v_e^{ON}(0) &= \frac{C_e v_e^{ON}(DT_s) - C_i v_i^{ON}(DT_s)}{C_e + C_i} e^{-(1-D)/f_s \tau_{OFF}} \\ &= \frac{R_e}{R + R_i + R_e} V_{dc} + k_1 + k_2 \end{aligned} \quad (38)$$

Following a similar line of reasoning for  $v_i$ , we find:

$$\begin{aligned} v_i^{ON}(0) &= \frac{C_i v_i^{ON}(DT_s) - C_e v_e^{ON}(DT_s)}{C_e + C_i} e^{-(1-D)/f_s \tau_{OFF}} \\ &= \frac{R_i}{R + R_i + R_e} V_{dc} + \left( \frac{RC_e}{\tau_1} - \frac{R + R_e}{R_e} \right) k_1 + \left( \frac{RC_e}{\tau_2} - \frac{R + R_e}{R_e} \right) k_2 \end{aligned} \quad (39)$$

In these expressions,  $v_e$  and  $v_i$  are given by Eqs. (22) and (24), respectively.

We now have two equations in the two unknown coefficients  $k_1$  and  $k_2$ . These are solved in a straight forward manner, but since the actual algebraic expressions that result are rather large, it is left to the interested reader to derive them. It should be noted that the constants are functions of the switching frequency  $f_s$  and the duty ratio  $D$ . With  $k_1$  and  $k_2$  in place the full expressions for the voltages in the on-period have been found, and this in turn gives us the initial value for  $v_e$  in the off-period, see Eq. (34). Thus, a complete solution for the steady-state has been derived, including the possibility of varying the duty ratio (or mark space ratio).

## 5. A discussion of the “charge hypothesis”

### 5.1. The charge hypothesis

Bailes found that the mean square of the variable part of  $Q_{e-i}$  in the off-period had a distinct maximum at the same switching frequency that gave optimum coalescer performance [10]. He also showed that the mean square had a remarkable similarity with his experimentally determined coalescence parameter at all frequencies that were tested at a duty ratio of 0.5. He formed a hypothesis as to why this happened: charge separation takes place during the on-period with negative charge accumulating at the emulsion–insulation interface and positive charge at the lower electrode (in his case bulk water); then, during the off-period, “depending on whether the drops are near the top or bottom of the oil they pick up negative or positive charges. Since the drops are mobile, the attraction between unlike charges then causes drop–drop coalescence and the formation of large drops which rapidly gravitate to the bulk interface” [10].

Experimental work that has been performed more recently, show that water droplets that come in contact with electrodes with non-zero potential are indeed likely to get charged [5]. A charged equipotential surface, such as the emulsion–insulation interface, is – from a macroscopic electromagnetic point of view – equivalent to a charged electrode. However, the system used by Bailes was not a symmetrical system as far as charges are concerned; considerably more negative charge would accumulate at the emulsion–insulation interface than at the emulsion-free water interface. (This can be derived from the analysis of Section 3.) From this it follows that even though droplets near the interfaces may get charged, one would expect considerably more negative drops resulting from this phenomenon than positive. Thus, much of the negative charge would be inactive in the coalescence process following Bailes’ argument.

His theory may still be valid, but if so, one may ask whether it would be better to make a symmetrical coalescer—one that actually results in equal amounts of positive and negative free charge. This could be done by placing the electrodes vertically instead of horizontally, and insulating both of them equally. Then a symmetrical situation would exist and an equal amount of positive and negative charge would be available. This would probably require that free water was removed before the coalescer-step, as, otherwise, the emulsion would be short circuited by the water.

However, there is also the possibility that the asymmetry does not matter much. One can still envisage that the drops near the emulsion–insulation interface acquire negative charge. Due to the electric field, which in the off-period is directed from the ground electrode (below) towards the emulsion–insulation interface, the charged droplets will move in direction negative electrode. While moving, they may collide with neutral (or positive) drops in their way, and coalescence could occur.

Another issue that was not mentioned in the paper by Bailes, is that there are actually two other physical parameters that behave exactly like the charge in the off-period. As shown below, these are, in addition to the charge, the voltage across the emulsion and the conductive current through it. Consequently, they all show the same correlation with coalescer performance.

### 5.2. Behavior of charge, voltage and current in the off-period

Eqs. (6) and (27) combined show that the free charge at the emulsion–insulation interface can be written, in the off-period, as

$$Q_{e-i} = (C_i + C_e)v_e \quad (40)$$

From this and Eq. (29) it is immediately clear that

$$\frac{dQ_{e-i}}{dt} = -\frac{Q_{e-i}}{\tau_{OFF}} \quad (41)$$

By definition, the time derivative of the free charge equals the net conductive current flowing into the interface, such that

$$\frac{dQ_{e-i}}{dt} = i_{R_i} - i_{R_e} \quad (42)$$

where  $i_{R_i}$  and  $i_{R_e}$  are the currents through the resistances  $R_i$  and  $R_e$ , respectively. Since, in the off-period,

$$i_{R_i} = \frac{v_i}{R_i} = -\frac{v_e}{R_i} = -\frac{R_e}{R_i} i_{R_e} \quad (43)$$

it follows that

$$i_{R_e} = -\frac{R_i}{R_e + R_i} \frac{dQ_{e-i}}{dt} = \frac{R_i}{R_e + R_i} \frac{Q_{e-i}}{\tau_{OFF}} \quad (44)$$

It is then further easily shown that

$$\frac{di_{R_e}}{dt} = -\frac{i_{R_e}}{\tau_{OFF}} \quad (45)$$

The differential equation for the emulsion voltage has been derived previously, Eq. (29), and we can now conclude that the voltage over the emulsion, the interfacial free charge and the conductive current in the emulsion all decay in the same manner during the off-period: exponentially with time constant  $\tau_{OFF}$ . They also have initial values that are proportional. The initial value for  $v_e$  has already been found and is given by Eq. (34). The numerator of this equation is nothing but the initial value of the interfacial free charge in the off-period whereas the denominator is a constant; thus the voltage and the charge are proportional. The initial value of  $i_{R_e}$  is also proportional to the initial value of  $v_e$  since  $i_{R_e}$  is always proportional to  $v_e$  through the law of Ohm. The latter fact could of course have been used to arrive at Eq. (45) more directly; however, its alternative derivation does in itself give some physical insight and is therefore used above.

On this basis, it is clear that the mean square of (the variable part) in the off-period of all these quantities have maximum value at the same frequency (and duty ratio). This could open up for the possibility of alternative hypotheses to explain the experimental data of Bailes.

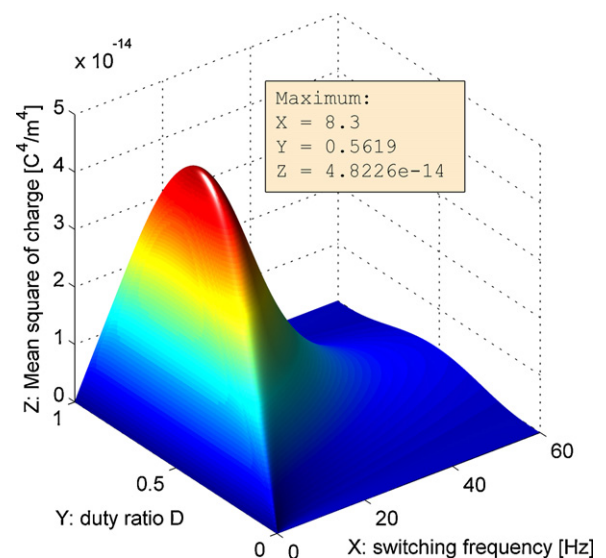


Fig. 5. Mean square of the variable part of the interfacial charge density as a function of switching frequency and duty ratio with parameters derived from Bailes.

### 5.3. Calculation of the mean square

Any of the three quantities discussed above can, as shown, be written as  $Ae^{-t/\tau_{OFF}}$  in the off-period, where  $A$  is a (negative) constant. At the end of the off-period, there will still be some residual interfacial charge, voltage or conduction current in the emulsion, which must equal  $Ae^{-t_{off}/\tau_{OFF}}$ . When Bailes found his correlation between the mean square of the charge and coalescer performance, he subtracted this residual part before he calculated the mean square. This is what is meant when it is referred to “the variable part of” in the text above. (Bailes called it “available charge density” [10].) Following this procedure, the mean square is:

$$\frac{1}{T_s} \int_0^{t_{off}} (Ae^{-t/\tau_{OFF}} - Ae^{-t_{off}/\tau_{OFF}})^2 dt$$

$$= \frac{A^2 \tau_{OFF} f_s}{2} (e^{-2(1-D)/\tau_{OFF} f_s} - 1) + 2A^2 f_s \tau_{OFF} (e^{-2(1-D)/\tau_{OFF} f_s} - e^{-(1-D)/\tau_{OFF} f_s}) + A^2 (1-D) e^{-2(1-D)/\tau_{OFF} f_s} \quad (46)$$

As can be seen, the mean square is a function of the switching frequency and duty ratio.

## 6. Numerical results

Parameters derived from Bailes are used [10]. These are  $V_{dc} = 200$  V,  $R = 100$  M $\Omega$ ,  $R_i = 600$  G $\Omega$ ,  $R_e = 79$  M $\Omega$ ,  $C_i = 163$  pF, and  $C_e = 37$  pF. With these particular parameters, Bailes' experimentally found optimum frequency was 8 Hz. He also found that the best duty ratio was 0.5, but this was tested with a switching frequency of 50 Hz.

In Fig. 5 the mean square given by Eq. (46) is plotted as a function of switching frequency and duty ratio. To enable a direct comparison with Bailes' numbers, the mean square of the charge density is plotted. For this purpose an electrode area of 0.0315 m<sup>2</sup>, which can also be derived from Bailes' paper, was used. Also shown in the figure is the coordinate of the maximum point of this graph. As can be seen, the optimum duty ratio (using the mean square criterion) is actually 0.56, but the difference from a duty ratio of 0.5 is small.

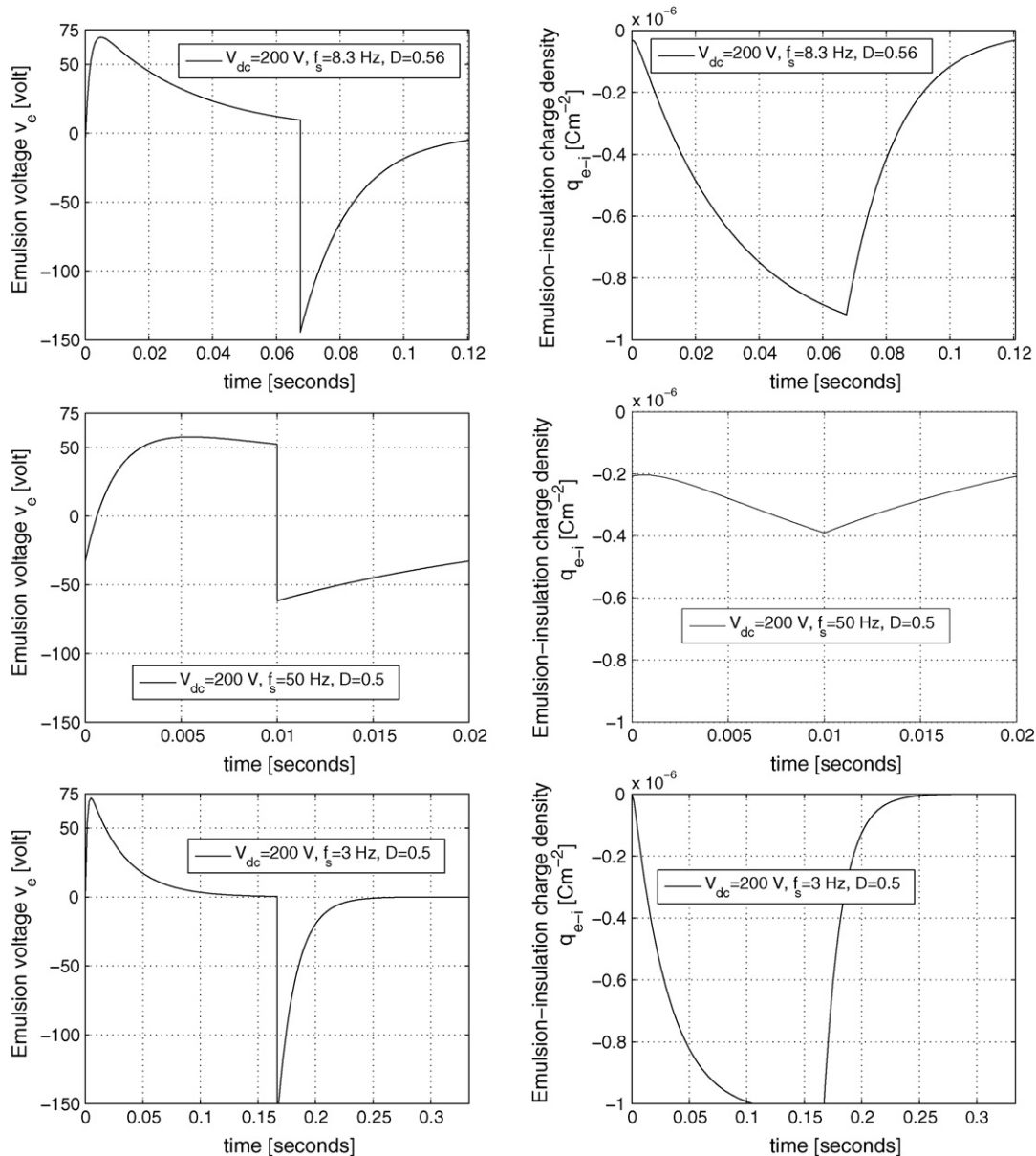


Fig. 6. Emulsion voltage and interfacial free charge density as functions of time for three different combinations of frequencies and duty ratio.

At 50 Hz, the frequency where Bailes experimented with different duty ratios, the optimum is indeed very close to 0.5.

Fig. 6 shows the periodic voltage across the emulsion ( $v_e$ ) for the maximum point of the graph in Fig. 5, i.e. for a frequency of 8.3 Hz and a duty ratio of 0.56. Also shown is the interfacial free charge. The same quantities are calculated for a switching frequency of 50 Hz and duty ratio of 0.5, as well as for a switching frequency of 3 Hz and duty ratio 0.5.

## 7. Discussion

These pictures clearly illustrate that even though the coalescer device itself is short circuited, this does not mean that the electric field (voltage) across the emulsion is zero. However, if the switching frequency is too low, the voltage would be (almost) zero in large parts of both the on- and off-periods, and the system would be inefficient. On the other hand, if the frequency is too high, the electric field strength may be relatively constant in value, but never comparable in magnitude to the voltage of the source, and the charge never builds up to the value that is achieved when the frequency and duty ratio is “just right”.

For the latter situation, the dynamics of the system is such that the magnitude of voltage experienced by the emulsion is comparable to the voltage of the source, and at the same time there is considerable variation in the strength over the period. Besides, the charge builds up to a significant value, and the coalescer discharges in just the right ratio, such that the charge starts at almost zero (but not quite) when the cycle repeats itself.

When there is a large variation over time of the electric parameters of the oil that is subjected to the coalescer, one would – on this basis – also expect a large variation of the performance of such a system.

The picture of the mean square confirms the earlier calculations given by Bailes, both in values and shape of the graph as a function of frequency. However, the expanded analysis presented here also includes the variation with duty ratio, and shows that a duty ratio of slightly more than 0.5 could have given even better coalescer performance at 8 Hz for his system.

The only measure of success for an electrostatic coalescer device is that it actually increases the separation of water from the emulsion. It should be emphasized that the results presented here cannot be used to predict actual coalescer performance. Ultimately, it would be a goal to be able to calculate the increased water mass flux resulting from subjecting an emulsion with known properties to a specific electric field profile. However, a necessary step needed for arriving at such a deep level of understanding, is to be able to predict the electric field that the emulsion actually experiences from an applied voltage profile over the device. The contribution of this paper is that it addresses this question.

## 8. Conclusions

This paper has re-examined and expanded the mathematical analysis of the electric field in a coalescer subjected to a pulsed

dc voltage. Emphasis was put on a physical understanding of the electric field and the build-up of charge in the system, using both electrostatic field theory as well as circuit analysis. A complete dynamic solution was derived; the formulas developed give the periodic steady-state solution for any frequency and duty ratio.

The physical arguments indicate that there must be an optimum frequency (or frequency range) when a pulsed dc scheme is used. This is further confirmed by looking at the electric field resulting from circuit analysis. Thus, a system that is designed for a particular oil, may fail to operate as expected when subjected to another oil with different material properties. Therefore, a tuning of the frequency should be a possibility in such systems.

The further evolution of commercial in-line coalescers are probably hampered by a lack of understanding of both the micro-scale particle interactions and a lack of understanding of the electric field that is actually experienced by the emulsion and thereby the dispersed water droplets, which – as we have seen – may be very different from the voltage applied on the terminals of the system.

It is hoped that the analysis presented in this paper could contribute to a better understanding of the importance of this aspect in electrostatic coalescer design. The analysis presented gives important physical insight, but is limited in practical design in that the geometry assumed was such that discrete parameters could be used. With more complicated geometrical structures, analytical solutions to the field problem may not be found, but approximate models consisting of networks of lossy dielectrics can be derived. Such models can be analyzed numerically in circuit simulators if the analytical approach becomes too cumbersome. In addition, to optimize the design further in terms of the electric field, software based on the finite element method can give good answers even for complicated geometries.

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